Eliminations of Some Defects of the Digital Cameras Used in the Laser Scanning Systems

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Key words: distortion, calibration, Alltran, inner orientation, outer orientation

SUMMARY

Two variables power series suitability analysis for description of defects of the digital cameras is presented. A large number of series is considered, from those currently used in photogrammetry up to series untypical in this area. The series are applied by adjusting in the single step along with a projection function or they are applied separately. From the projection functions we consider the collinear transformation and the projective transformation. We solve algorithmization of the stated transformations and power series and their implementation into the Alltran library. The most suitable solution will be applied for mapping of textures in the PointClouder software and as a new mathematical base for intersection in the LORS system.
1. INTRODUCTION

Digital camera is an important part of 3D scanning systems. It is used directly for determination of 3D coordinates in the case of scanning systems based on spatial forward intersection from angles. In the case of the scanning systems based on the spatial polar method a digital camera is used for the determination of scanning area and/or for coloring of measured points by RGB colors. The special case of digital camera using is so called "3D Image" – an image which has 3D coordinates assigned to each pixel.

It is necessary to know the mathematical relation between image and spatial coordinates in all stated cases. The relation is composited from the central projection and function which describe all other influences like objective distortion, image sensor surface distortion, sensor turning to objective optical axis and others. The first relation is exactly known, but the second one depends on a various factors and it is possible to use a long range of function to describe it.

The main method for usability assessing of used function is its application on control points which were not used for computation of its parameters and computation of the positional standard deviation in image coordinates. The another method for usability assessing is application of inverse function on the control points read image coordinates of another image and its using for computation of the central projection. The unit standard deviation of the central projection should be influenced only by measuring errors in this case.

The additional method is solution stability assessing. The stability can be assessed by the maximal correlation among adjusted parameters or by the condition number in the case of the singular value decomposition solution.

2. MATHEMATICAL RELATIONS

2.1 Central Projection

The basic photogrammetric relations for central projection were used.

2.1.1 Projective Transformation

The main used function is the projective transformation:
\[
x = x_0 - f r_{11} \cdot (X - X_0) + r_{21} \cdot (Y - Y_0) + r_{31} \cdot (Z - Z_0) \\
y = y_0 - f r_{12} \cdot (X - X_0) + r_{22} \cdot (Y - Y_0) + r_{32} \cdot (Z - Z_0)
\]

where \(x, y\) are image coordinates, \(X, Y, Z\) are spatial coordinates, \(x_0, y_0\) are coordinates of the principal point, \(X_0, Y_0, Z_0\) are spatial coordinates of the projection center, \(f\) is the focal length and from \(r_{11}\) to \(r_{33}\) are the elements of rotation matrix.

If we don't use rotation angles, but elements of rotation matrix, then it is necessary to use orthogonality constraints:

\[
r_{11}^2 + r_{12}^2 + r_{13}^2 - 1 = 0,
\]
\[
r_{21}^2 + r_{22}^2 + r_{23}^2 - 1 = 0,
\]
\[
r_{31}^2 + r_{32}^2 + r_{33}^2 - 1 = 0,
\]
\[
r_{11} \cdot r_{21} + r_{12} \cdot r_{22} + r_{13} \cdot r_{23} = 0,
\]
\[
r_{21} \cdot r_{31} + r_{22} \cdot r_{32} + r_{23} \cdot r_{33} = 0,
\]
\[
r_{11} \cdot r_{31} + r_{12} \cdot r_{32} + r_{13} \cdot r_{33} = 0.
\]

This transformation describes central projection from the object space to the image plane in the case that the principal point is the nearest image point to projection center. The inner orientation elements \((x_0, y_0, f)\) is usually known. If we don't know the inner orientation elements we can determine some of them as adjusted parameters. It is not possible to determine all three elements of inner orientation from one single image of the planar calibration field.

The projection transformation is the only one which is not linear from adjusted parameters view. The biggest problem is solving of approximate values of searched parameters. We have found such solution in publication [1] and it has been implemented into Alltran library (more in chapter 3.2). It is better to use original solution suggested by one of the paper authors for the planar calibration field (see reference [2]). This solution uses all control points for solving approximate numbers and that's why they are more accurate.

### 2.1.2 Direct Linear Transformation in 2D

The second used central projection transformation is so called colinear transformation (in some literature referenced as Direct Linear Transformation in 2D – further DLT2D):
The transformation describes central projection between planes. It has more degrees of freedom than the projective transformation. More information about the basic photogrammetry transformation can be found in references [1] and [3].

$$x = \frac{L_1 \cdot X + L_2 \cdot Y + L_3}{L_7 \cdot X + L_8 \cdot Y + 1}$$
$$y = \frac{L_4 \cdot X + L_5 \cdot Y + L_6}{L_7 \cdot X + L_8 \cdot Y + 1}$$

(3)

The whole row of functions can be used for real camera defect description. The functions used in experiments are stated further. The functions can be applied in single step together with central projection or in a second step (separated adjustment).

### 2.2 The other influences

The basic function used together with central projection is following:

$$x = X(X, Y, Z) - (x - x_0)(k_1 \cdot r^2 + k_2 \cdot r^4 + k_3 \cdot r^6) - p_1 (r^2 + 2(x - x_0) y) - 2 p_2 (x - x_0)(y - y_0)$$
$$y = Y(X, Y, Z) - (y - y_0)(k_1 \cdot r^2 + k_2 \cdot r^4 + k_3 \cdot r^6) - p_1 (r^2 + 2(y - y_0) x) - 2 p_2 (x - x_0)(y - y_0)$$

(4)

$$r^2 = (x - x_0)^2 + (y - y_0)^2$$

The parameters $k_1$, $k_2$ and $k_3$ are used for description of the radial distortion and the parameters $p_1$ and $p_2$ are used for the tangential distortion.

The function is traditionally applied in the read image coordinates, so it is necessary to compute them iteratively for transformation from the object space to the image plane:

$$x_{i+1} = X - (x_i - x_0)(k_1 \cdot r^2 + k_2 \cdot r^4 + k_3 \cdot r^6) - p_1 (r^2 + 2(x_i - x_0) y_i) - 2 p_2 (x_i - x_0)(y_i - y_0)$$
$$y_{i+1} = Y - (y_i - y_0)(k_1 \cdot r^2 + k_2 \cdot r^4 + k_3 \cdot r^6) - p_1 (r^2 + 2(y_i - y_0) x_i) - 2 p_2 (x_i - x_0)(y_i - y_0)$$

(5)

It is sufficient to apply only two first parameters of radial distortion in the most cases of high quality objectives. You can find more information about these relations in the reference [4].

There were applied only parameters of the radial distortion or parameters of the radial distortion and $x_0$, $y_0$ (in this case $x_0$, $y_0$ are not the principal point coordinates but the center of radial distortion) or all stated parameters.

These functions are also applied separately in the second step.
2.2.2 The functions applied separately in the second step

The main function in this category is the polynomial function of two variables:

\[ x = a_1 + a_2 \cdot X + a_3 \cdot Y + a_4 \cdot X^2 + a_5 \cdot X \cdot Y + a_6 \cdot Y^2 \]
\[ + a_7 \cdot X^3 + a_8 \cdot X \cdot Y^2 + a_9 \cdot X^2 \cdot Y + a_{10} \cdot Y^3 \]
\[ + a_{11} \cdot X^4 + a_{12} \cdot X \cdot Y^3 + a_{13} \cdot X^2 \cdot Y^2 + a_{14} \cdot X^3 \cdot Y + a_{15} \cdot Y^4 \]
\[ y = b_1 + b_2 \cdot X + b_3 \cdot Y + b_4 \cdot X^2 + b_5 \cdot X \cdot Y + b_6 \cdot Y^2 \]
\[ + b_7 \cdot X^3 + b_8 \cdot X \cdot Y^2 + b_9 \cdot X^2 \cdot Y + b_{10} \cdot Y^3 \]
\[ + b_{11} \cdot X^4 + b_{12} \cdot X \cdot Y^3 + b_{13} \cdot X^2 \cdot Y^2 + b_{14} \cdot X^3 \cdot Y + b_{15} \cdot Y^4 \]

The polynomial functions (transformation) of second (quadratic), third (cubic) and forth (quartic) order were used. The parameters from \(a_1\) to \(a_6\) and from \(b_1\) to \(b_6\) are used for the quadratic transformation, from \(a_1\) to \(a_{10}\) and from \(b_1\) to \(b_{10}\) for the cubic transformation and all the stated parameters for the quartic transformation.

All of the listed polynomial transformations are linear from the point of view of searched parameters so the approximate values are not required.

3. EXPERIMENT DESCRIPTION

3.1 Testing field

The precise planar calibration field was created at first. The number of control (identical) points is 300 and the field size is about 100x70 centimeters. The shape of control points is black annular ring with the outer radius four millimeters and the inner radius one millimeter. The shape is suitable for reading by planimeter and also for reading from digital images (it is aimed at the annular ring or the inner circle depending on image resolution). The calibration field was measured two times with double reading by planimeter made by Altec Corporation. The accuracy of one measuring is determined from two measuring differences and the standard deviation in both coordinates is approximately 0.04 millimeter (the standard deviation of average value is about 0.03 millimeter which corresponds to approximately 0.1 pixel on the used images).
Then a few pictures were made by cameras Canon EOS D350 (reflex camera, eight megapixels) with objective Tamron AF SP 28-75 (f2.8) with variable focal length. The tested images were taken with the edge position of the focal length \( f=28mm \). Image coordinates of control points were two times read by the software "odecitacv2" which was made by Ing. M. Štroner, Ph.D. This software enables a sub pixel selection of points by a method of RGB filter. After clicking into a pixel fulfilling the set filter, all neighboring pixels fulfilling the RGB filter are automatically selected and their average is calculated. The standard deviation of read coordinates was determined from multiple points reading. It is in the interval from 0.05 to 0.2 pixels depending on the used image.

### 3.2 The library Alltran

Alltran [5] is a library of classes and function for computation of transformation key and for transformation of coordinates. It was suggested especially for transformations based on the least square method. All included transformations are in close relation with the branch of geodesy and photogrammetry. The library is written in C++ language under the condition of General Public Licence GNU. The simple console program Alltran_console was created for using the library and another one Alltran_test for testing of the suitability of used transformations.

All stated transformations were implemented into the Alltran library.
3.3 Testing on one image

The main criterion for usability assessing of used function is its application on a part of control points which were not used for computation of its parameters and then computation of positional standard deviation in image coordinates:

\[
\sigma_p = \sqrt{\frac{\sum_{i=1}^{n} \left( (x_i - x_{iT})^2 + (y_i - y_{iT})^2 \right)}{n-1}},
\]

where \(n\) is the number of control points used for testing, \(x\) and \(y\) are the read image coordinates of the control points and \(x_{iT}\) and \(y_{iT}\) are their transformed coordinates.

The stated results were got from the image with 194 control points. The 97 points were three times randomly chosen and used for computing of the transformation keys and the rest 97 points were used for computing of the positional standard deviation. There were given more test and results in the reference [6], but all the results were in accordance with the stated.

### 3.3.1 Transformations based on DLT2D

There were given the results of following transformation in the reference [6] – tab.1.

<table>
<thead>
<tr>
<th>Tab. 1 – DLT2D based transformation testing</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Single step transformations</strong></td>
</tr>
<tr>
<td>dlt_2d</td>
</tr>
<tr>
<td>dlt_2d rd</td>
</tr>
<tr>
<td>dlt_2d rd2</td>
</tr>
<tr>
<td><strong>Two steps transformations</strong></td>
</tr>
<tr>
<td>dlt_2d+tps_2d</td>
</tr>
<tr>
<td>dlt_2d rd2+tps_2d</td>
</tr>
<tr>
<td>dlt_2d+rd</td>
</tr>
<tr>
<td>dlt_2d+rd2</td>
</tr>
<tr>
<td>dlt_2d+quadratic_2d</td>
</tr>
<tr>
<td>dlt_2d+cubic_2d</td>
</tr>
<tr>
<td>dlt_2d+quartic_2d</td>
</tr>
<tr>
<td><strong>New tested transformations</strong></td>
</tr>
<tr>
<td>dlt_2d rd td</td>
</tr>
<tr>
<td>dlt_2d+rd td</td>
</tr>
</tbody>
</table>

The names of transformation listed in the table 1 are based on the names of the transformations in the library Alltran. If there is the symbol "+" in the name, it means that the transformation is composed from two separately adjusted transformations.

The description of transformation dlt_2d is in the chapter 2.1.2, "rd" means the radial distortion described by parameters \(k_1\), \(k_2\) and \(k_3\), "rd2" adds to "rd" parameters \(x_0\) and \(y_0\), "rd_td" uses the all searched parameters of equations (4). "tps_2d" is "Thin Plate Splines"
transformation. "tps_2d" is pretty complicated, it is not based on least square adjustment and it gives bad results for lower numbers of control points (see reference [6]), so it wasn't used in further experiments. The names "quadratic_2d", "cubic_2d" and "quartic_2d" denote polynomial transformations described in the chapter 2.2.2.

3.3.2 Transformations based on the projective transformation

After implementing of the solution for computation of the projective transformation approximate values to the library Alltran the similar testing to DLT2D was possible. The projective transformation has much more practical application than DLT2D. It can be used e.g. for determination of the inner and outer camera orientation elements for some systems ([7] and [8]), a calculation of image coordinates for mapping textures onto a triangular mesh in the GUI (Graphic User Interface) PointClouder [10], for adding quality information about color of the measured points of the Leica HDS3000 scanning system and for computation of 3D Image [9].

We used Direct Linear Transformation (in 3D) for stated purposes at our working group in the past. Its main advantage is its linear form from adjusted parameters view and the main disadvantage is that its control points can't lie in the plane or near the plane, whereas creation and maintenance of a plane calibration field of control points is simpler than in the case of a spatial calibration field.

The listed transformations were tested (tab. 2):

<table>
<thead>
<tr>
<th>Number of control points for key computation</th>
<th>Single step transformation</th>
<th>$\sigma_p$ [pixel]</th>
</tr>
</thead>
<tbody>
<tr>
<td>projective</td>
<td></td>
<td>7.1 7.2</td>
</tr>
<tr>
<td>projective_x0y0</td>
<td></td>
<td>4.9 5.4</td>
</tr>
<tr>
<td>projective_f</td>
<td></td>
<td>5.3 5.3</td>
</tr>
<tr>
<td>projective_rd2</td>
<td></td>
<td>4.3 4.8</td>
</tr>
<tr>
<td>projective_x0y0_rd</td>
<td></td>
<td>1.5 1.6</td>
</tr>
<tr>
<td>projective_x0y0_rd_td</td>
<td></td>
<td>0.8 0.8</td>
</tr>
<tr>
<td>Two steps transformation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>projective_x0y0_plus_rd</td>
<td></td>
<td>2.7 3.6</td>
</tr>
<tr>
<td>projective_x0y0_plus_rd_td</td>
<td></td>
<td>2.6 3.2</td>
</tr>
<tr>
<td>projective_x0y0_plus_rd2</td>
<td></td>
<td>2.4 4.1</td>
</tr>
<tr>
<td>projective_x0y0_plus_quadratic_2d</td>
<td></td>
<td>4.8 5.5</td>
</tr>
<tr>
<td>projective_x0y0_plus_cubic_2d</td>
<td></td>
<td>0.6 0.7</td>
</tr>
<tr>
<td>projective_x0y0_plus_quartic_2d</td>
<td></td>
<td>0.6 1.0</td>
</tr>
<tr>
<td>projective_plus_cubic_2d</td>
<td></td>
<td>0.6</td>
</tr>
</tbody>
</table>

The tests were performed three times with randomly chosen points and the stated values are the averages.
"projective" is projective transformation (see chapter 2.1.1) with the approximate values of the inner orientation elements ($x_0$, $y_0$ and $f$) considered as fixed. In the case of "projective_x0y0" is fixed only the focal length (from inner orientation elements) and in the case of "projective_f" is fixed the principal point ($x_0$, $y_0$). The special case is "projective_rd2". It is projective transformation together with the standard function for radial distortion with the adjustable center of the radial distortion. The transformation "projective_x0y0_rd" is like "projective_x0y0" together with the radial distortion function. The point $x_0$, $y_0$ is the adjustable principal point and also the center of the radial distortion. The transformation "projective_x0y0_rd_td" is the same as previous one, but it has also parameters $p_1$ and $p_2$ which describes the tangential distortion. All other listed transformations are two steps transformations which are composed from the already described single step transformations.

### 3.4 Testing in two images manner

The another method for usability assessing is application of the inverse function on control points image coordinates of another image and then using of corrected coordinates for computation of the central projection. The unit standard deviation of the central projection should be influenced only by measuring errors in this case.

All the images were taken with the exactly same camera settings (fixed focus and focal length).

The best transformations from one image tests were considered. It means "projective_x0y0_rd_td", "projective_plus_cubic_2d", "projective_x0y0_plus_cubic_2d", "dlt_2d_rd_td" and "dlt_2d_plus_cubic_2d".

At first the transformation parameters on the calibration image (3182) were computed. The important parameters are $x_0$, $y_0$, $k_1$, $k_2$, $k_3$, $p_1$ and $p_2$ for the first transformation ("projective_x0y0_rd_td"), the parameters of the cubic transformation for the second one ("projective_plus_cubic_2d"), $x_0$, $y_0$ and the cubic transformation parameters for "projective_x0y0_plus_cubic_2d" and the similar groups of parameters for transformations based on DLT2D.

In the second step the inverse transformations to "rd_td" and cubic transformation were applied to read image coordinates of control points on other images (3180 a 3184). The inversion is simple in the case of "rd_td" because the transformation is defined in the coordinate system of original image coordinates (and during forward transformation is computed iteratively). The inversion of the cubic transformation is complicated because it is not possible to directly invert its original form (6). That's why the iterative inversion was used:

\[
\begin{align*}
X_{i+1} = X_i + x - f(X_i,Y_i) \\
Y_{i+1} = Y_i + y - g(X_i,Y_i)
\end{align*}
\]  

In the third step the central projection transformations are applied on corrected image coordinates and their unit standard deviation is assessed. The unit standard deviation has the same meaning like the position standard deviation (see equation (7)).
### CONCLUSION

The various transformations testing for the elimination of the digital cameras defects is presented in the paper. The most suitable solution seems to be the cubic transformation. It is linear that's why it is simply computable and applicable. It needs only a small number of control points to high quality description of common digital cameras defects. The solution is also numerically stable. The cubic transformation is suitable for both the projective and the collinear (DLT2D) transformation.

*This research has been supported by GA ČR grant No. 103/06/0094.*

### REFERENCES


**BIOGRAPHICAL NOTES**

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