

Terrestrial Laser Scanning

3. Lecture

DTM, triangulation, SLAM

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Contents

- Digital Terrain Model
- Delaunay triangulation 2D a 3D
- SLAM algorithm

Digital Terrain Model

- A Digital Terrain Model (DTM) is a mathematical representation (model) of the ground surface.
- The necessary data are acquired by classical geodetic methods (tacheometric measurements with total stations or GNSS receivers) and currently mainly by Airborne laser scanning (ALS) and terrestrial scanning (kinematic and static (smaller areas))
- A DTM is expressed:
 - in the form of a regular grid, in which a unique elevation is assigned to each pixel.
 - In the form of a triangulated irregular network (TIN)

Digital Terrain Model - Other terrain models

- Digital Elevation Model (DEM) represents the bare-Earth surface, removing all natural and built features
- Digital Surface Model (DSM) captures both the natural and built/artificial features of the environment
- A DTM typically augments a DEM, by including vector features of the natural terrain, such as rivers and ridges. A DTM may be interpolated to generate a DEM, but not vice versa.
- DEM is used in areas such as land-use planning, infrastructural project management, soil science, hydrology and flow-direction studies
- DSM is used in areas such as urban planning (investigating how a proposed building would affect the view), aviation planning or forestry management (height of trees – Canopy Height Model (CHM) – the difference between the DSM and DEM)

Source: [DEM, DSM & DTM: Digital Elevation Model - Why It's Important - AEVEX \(geodetics.com\)](https://www.geodetics.com/DEM-DSM-DTM-Digital-Elevation-Model-Why-Its-Important-AEVEX)

Mesh

- Composition to describe objects that cannot be expressed as geometric primitives (plane, sphere, cylindrical surface, cone, etc.).
- It is expressed in the form of irregular triangles in space – in general mesh is spatial (3D)
- Requirements for creation: that the triangles be as close as possible to equilateral.
- Algorithms based on Delaunay triangulation are used
- Mesh can also be expressed in the form TIN

Mesh - TIN

- TIN is a representation of a surface consisting entirely of triangles (triangle mesh)
- Used in many applications such as Geographic Information Systems (GIS), Computer Aided Design (CAD) for visual representation of topographic surface (DEM, DSM, DTM)
- **The TIN is formed by a network of non-overlapping triangles in the projection direction**
- **In general TIN is not full triangulation in 3D**
- It is so called 2,5D – points are projected from 3D space into 2D plane, there are solved triangles and then 3D coordinates are returned to the vertex of the triangles

Delaunay triangulation for TIN

- Boris Delaunay: Sur la sphère vide, Izvestia Akademii Nauk SSSR, Otdelenie Matematicheskikh i Estestvennykh Nauk, 7:793-800, 1934.

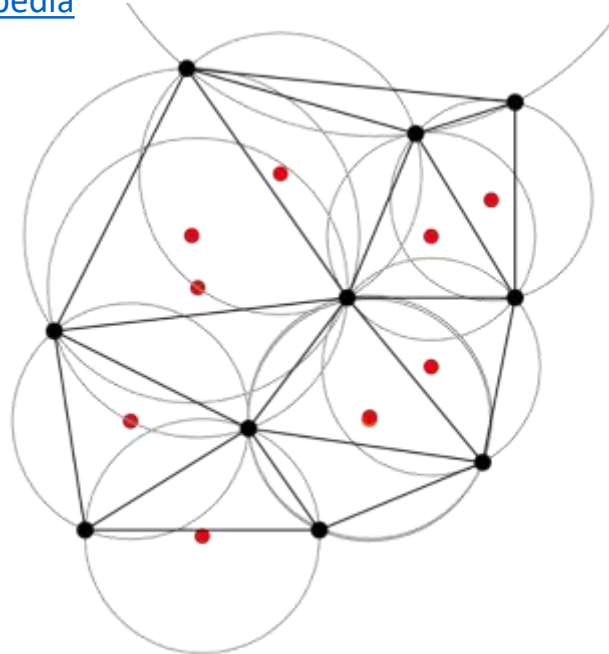
Principle:

- There must be no other point in the circle circumscribed by any triangle.
- Let P be the set of n points in the plane not lying on a line, and let k be the number of points that lie on the boundary of the convex hull of the points of the set P . Then:
- Every triangulation of P (i.e., even a Delaunay triangulation) has $2n-2-k$ triangles and $3n-3-k$ edges.
- The triangulation maximizes the minimum angle.
- It is the basis of the vast majority of automatic algorithms for constructing triangular networks, or algorithms satisfying its condition.

Delaunay triangulation

- a Delaunay triangulation for a given set P of discrete points in a general position is a triangulation $DT(P)$ such that no point in P is inside the circumcircle of any triangle in $DT(P)$. Delaunay triangulations maximize the minimum of all the angles of the triangles in the triangulation; they tend to avoid sliver triangles.

[Delaunay triangulation - Wikipedia](#)

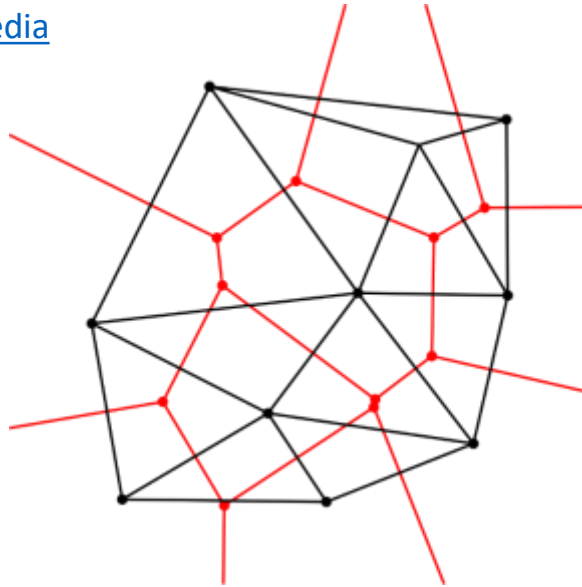


The Delaunay triangulation with all the circumcircles and their centers (in red).

Delaunay triangulation

- The Delaunay triangulation of a discrete point set P in general position corresponds to the dual graph of the Voronoi diagram for P . The circumcenters of Delaunay triangles are the vertices of the Voronoi diagram. In the 2D case, the Voronoi vertices are connected via edges, that can be derived from adjacency-relationships of the Delaunay triangles: If two triangles share an edge in the Delaunay triangulation, their circumcenters are to be connected with an edge in the Voronoi tessellation.

[Delaunay triangulation - Wikipedia](#)



Connecting the centers of the circumcircles produces the Voronoi diagram (in red).

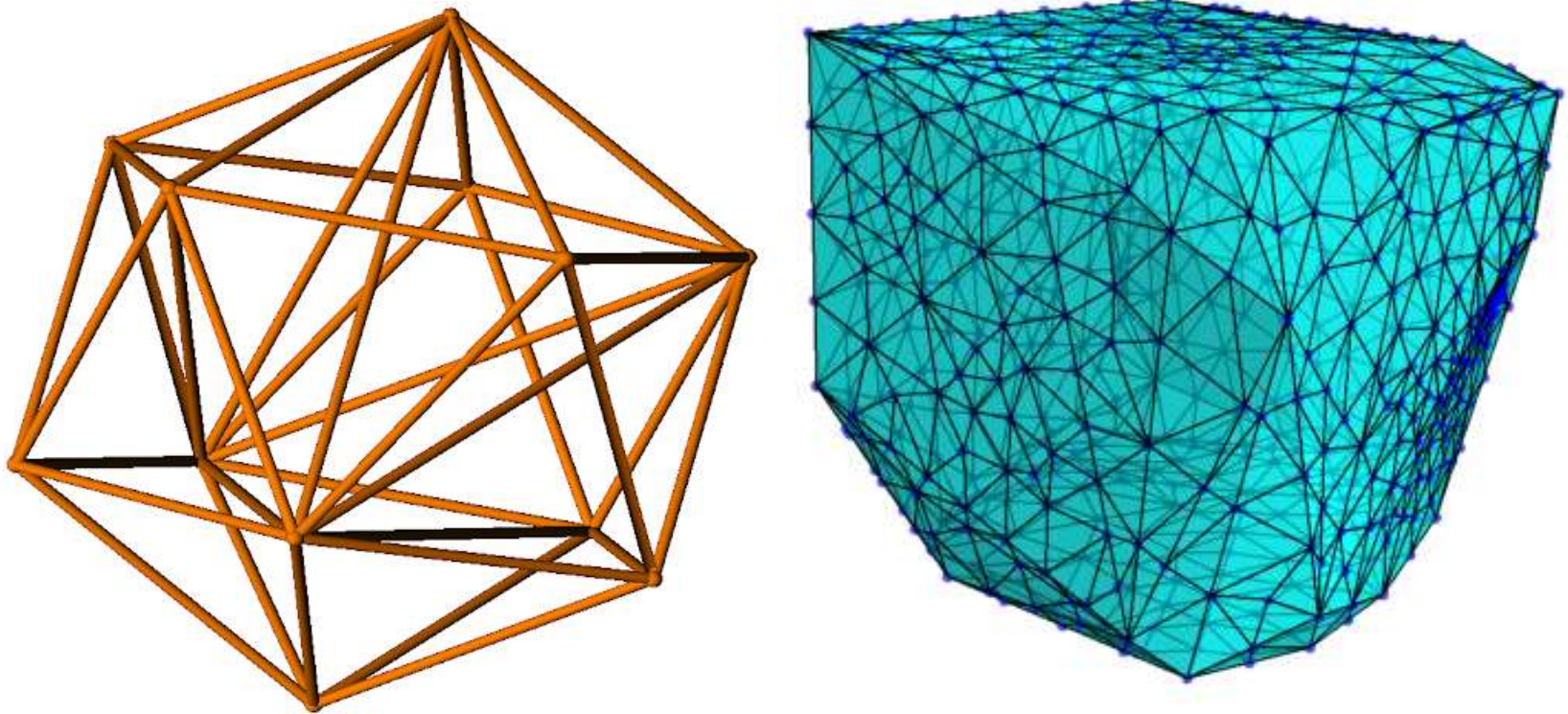
Delaunay triangulation / tetrahedronisation

- The Delaunay triangulation (like the Voronoi diagram) can be defined for any number of dimensions. The triangulation of a d -dimensional point set consists of adjacent d -simplexes such that the d -dimensional hypercube described by each simplex contains no point from the set inside.
- A d -simplex: a d -dimensional generalization of a triangle (a triangle can be referred to as a 2-simplex). For example, a 3-simplex is a tetrahedron, a 1-simplex is a line segment.
- Thus, in the case of a 3-dimensional space, it is a network of adjacent tetrahedra, where the sphere circumscribed to each tetrahedron must be empty. Similar to the 2D case, this triangulation is unambiguous if and only if no vertices other than the vertices of the tetrahedron lie on the surface of the sphere imprinted to each tetrahedron.

J. Zika: Algoritmus pro 3D Delaunayovu triangulaci. Bakalářská práce 2012.

Delaunay triangulation / tetrahedronisation

- Delaunay 3D mesh



http://www.preschern.org/detri/DeTri_en.html

<https://mathematica.stackexchange.com/questions/11358/delaunay-triangulation-for-3d-surface-data>

Meshing in Cyclone

External source:

- mikulas_zpracovani → ScanWorld 1_4 → kousek mračna

TIN meshing

X

Complex meshing

SLAM

- Solves the problem of movement in a partially or completely unknown environment
- Sequential creation of an environment map and simultaneous localization using the same map
- Use of robot navigation, mobile scanning
- A range of sensors: scanner, camera, compass, INS, ...
- Landmarks:
 - distinctive and easily distinguishable objects in the data
 - repeatably observable
 - in sufficient quantity
 - static

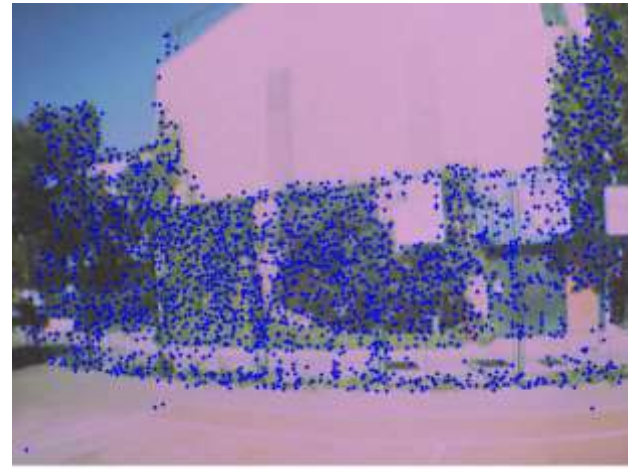
SLAM

- SLAM algorithm procedure:
 1. Inaccurate estimation of current position
 2. Ambient measurement (laser scanner)
 3. Find landmarks in the measurement
 4. Compare current landmarks with previously observed landmarks (stored in memory from previous step)
 5. Based on the comparison of the current and original landmarks, calculate the change in position of the device (scanner, robot) and update the position estimate
 6. Store the newly observed landmarks in memory

SLAM

- Identification of keypoints on images and resulting landmarks

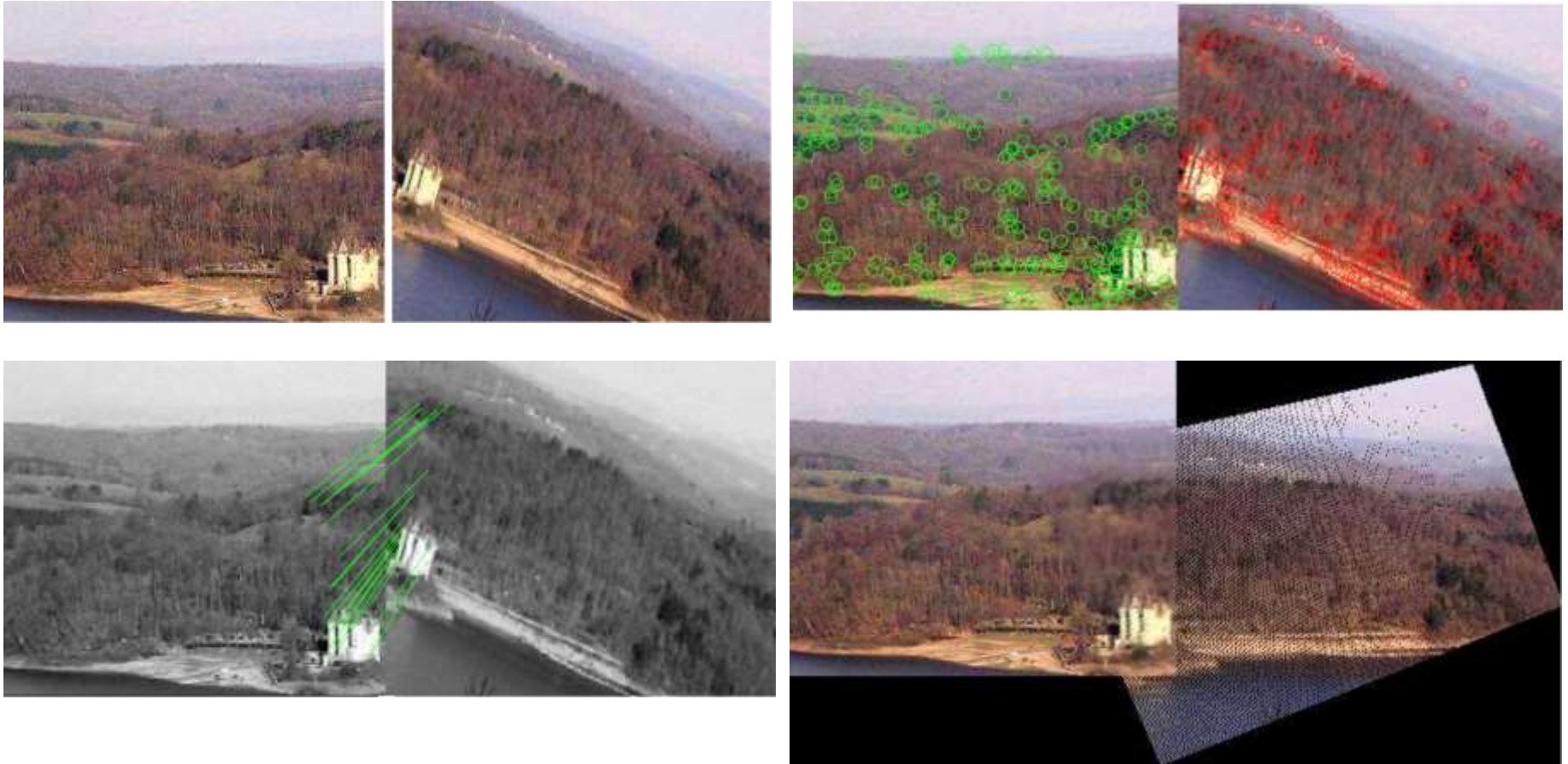
[Nemra Thesis](#) | [PDF](#) | [Unmanned Aerial Vehicle](#) | [Kalman Filter \(scribd.com\)](#)



SLAM

- Identification of keypoints on images and resulting landmarks

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Thank you for your attention!