Fieldwork Surveying FS01

2. Lecture

Basic Geodetic Calculation

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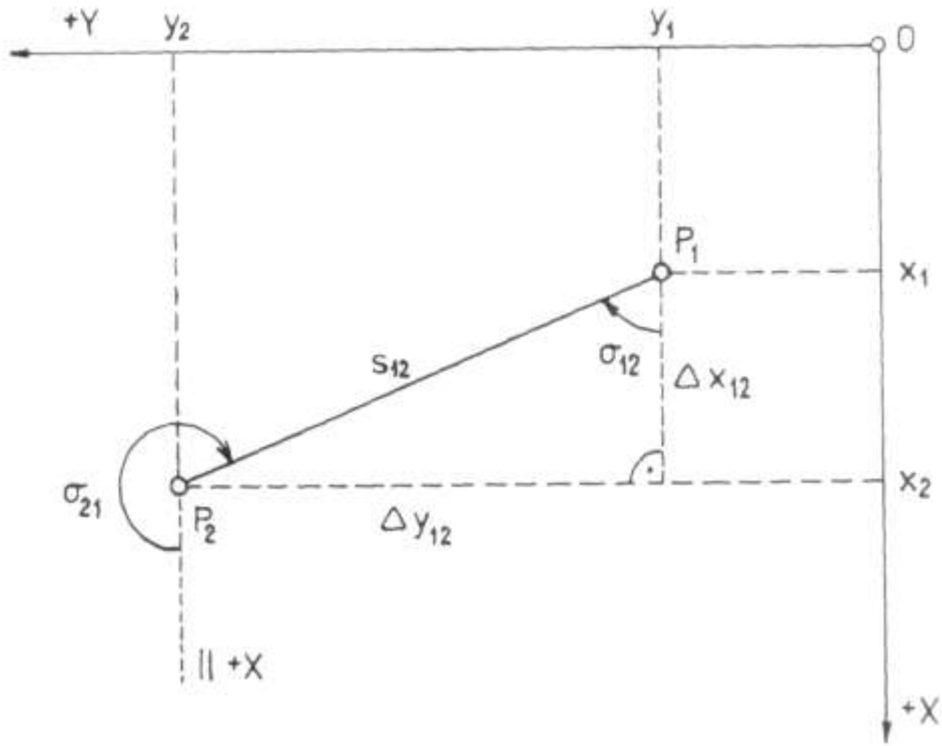
Coordinate system

- position of points is defined by **rectangular plane coordinates Y, X** in given coordinate system (reference frame)
- geodetic coordinate system S-JTSK is **clockwise**



coordinate differences

$\Delta x_{12} = x_2 - x_1$

$\Delta y_{12} = y_2 - y_1$

$\Delta x_{21} = x_1 - x_2$

$\Delta y_{21} = y_1 - y_2$

distance

$$s_{12} = \sqrt{\Delta x_{12}^2 + \Delta y_{12}^2}$$

$s_{12} = s_{21}$

$s_{12} = \Delta y_{12} / \sin \sigma_{12}$

$s_{12} = \Delta x_{12} / \cos \sigma_{12}$

Bearing

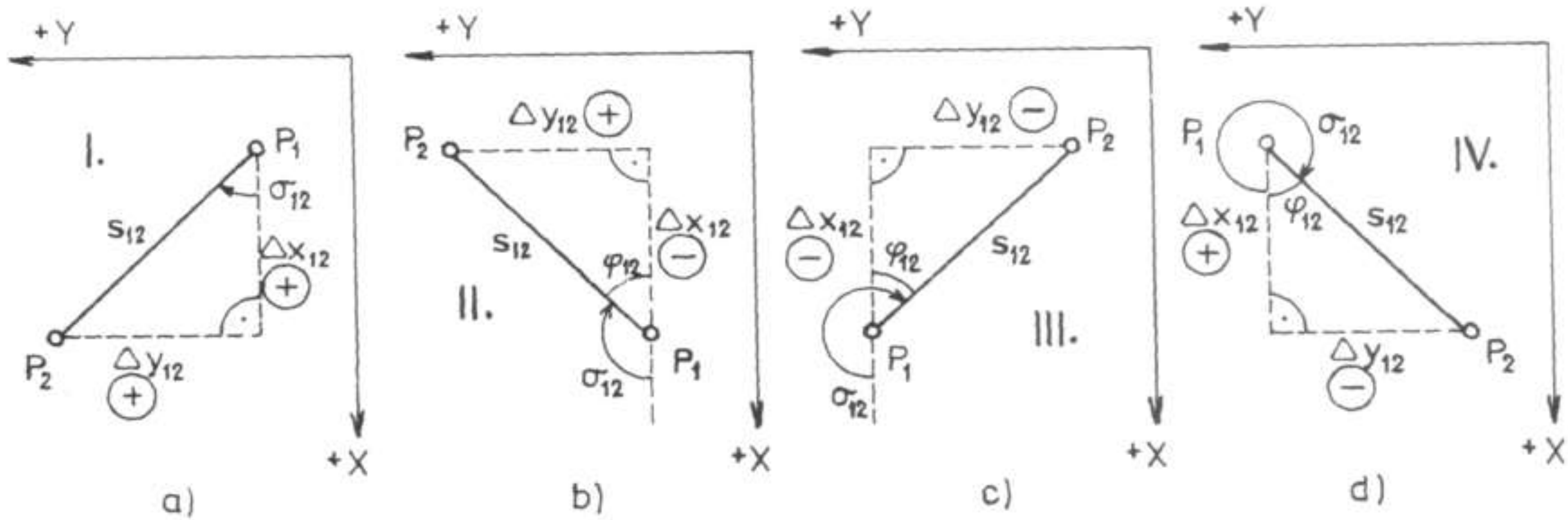
- oriented angle between parallel to the axis +X and the join of the points

$\sigma_{21} = \sigma_{12} + 180^\circ$

$\sigma_{21} = \sigma_{12} + 200 \text{ gon} = \sigma_{12} + 200^g$

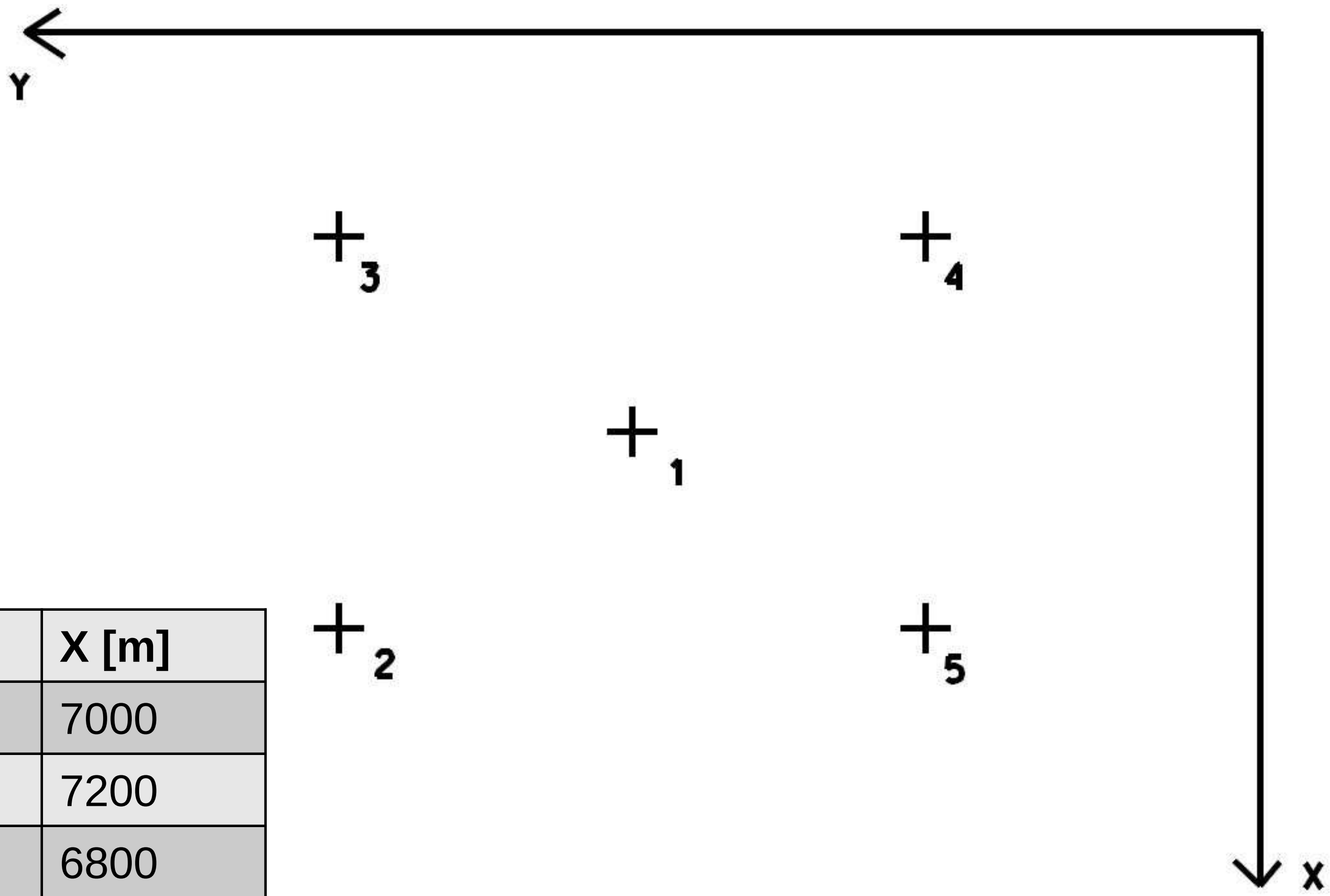
$$tg\ \varphi_{12} = \frac{|\Delta y_{12}|}{|\Delta x_{12}|}$$

Bearing



Quadrant	I	II	III	IV
Δy_{12}	+	+	-	-
Δx_{12}	+	-	-	+
	$\sigma_{12} = \varphi_{12}$	$\sigma_{12} = 200^g - \varphi_{12}$	$\sigma_{12} = 200^g + \varphi_{12}$	$\sigma_{12} = 400^g - \varphi_{12}$

Bearing – examples



P. N.	Y [m]	X [m]
1	2000	7000
2	2300	7200
3	2300	6800
4	1700	6800
5	1700	7200

Bearing – examples

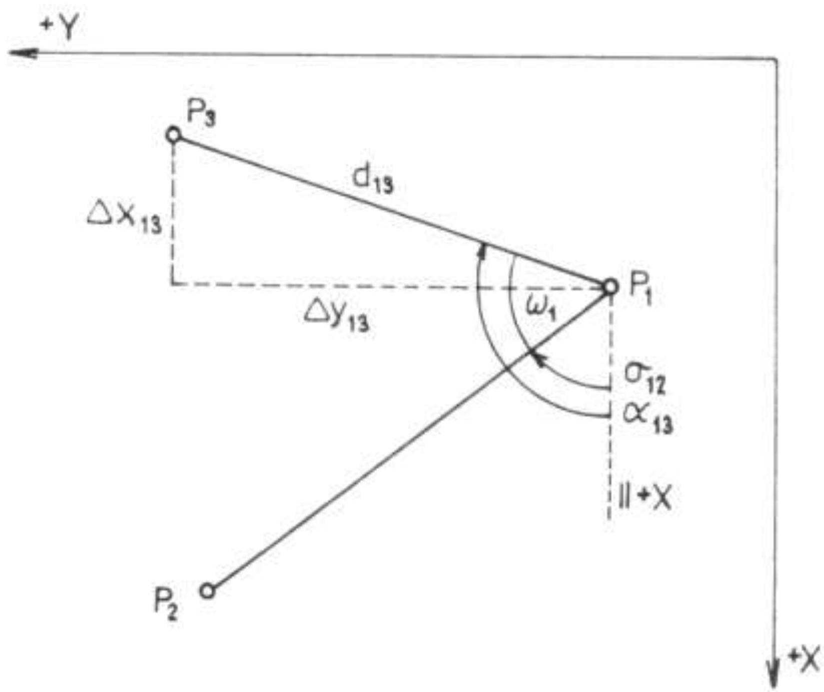
$$\varphi_{12} = \arctan\frac{|\Delta Y_{12}|}{|\Delta X_{12}|} = \arctan\frac{|+300|}{|+200|} = 62,5666 gon \qquad \sigma_{12} = \varphi_{12} = 62,5666 gon$$

$$\varphi_{13} = \arctan\frac{|\Delta Y_{13}|}{|\Delta X_{13}|} = \arctan\frac{|+300|}{|-200|} = 62,5666 gon \qquad \sigma_{13} = 200 gon - \varphi_{13} = 137,4334 gon$$

$$\varphi_{14} = \arctan\frac{|\Delta Y_{14}|}{|\Delta X_{14}|} = \arctan\frac{|-300|}{|-200|} = 62,5666 gon \qquad \sigma_{14} = 200 gon + \varphi_{14} = 262,5666 gon$$

$$\varphi_{15} = \arctan\frac{|\Delta Y_{15}|}{|\Delta X_{15}|} = \arctan\frac{|-300|}{|+200|} = 62,5666 gon \qquad \sigma_{15} = 400 gon - \varphi_{15} = 337,4334 gon$$

Determination of a point defined by polar coordinates (bearing and distance)



Given:

rectangular coordinates of points P_1 [y_1, x_1] and P_2 [y_2, x_2],

distance d_{13}

horizontal angle ω_1

Calculated: P_3 [y_3, x_3]

$$tg\ \varphi_{12} = \frac{|\Delta y_{12}|}{|\Delta x_{12}|}$$

according to the table $\rightarrow \sigma_{12}$

$\alpha_{13} = \sigma_{12} + \omega_1$

Coordinate differences:

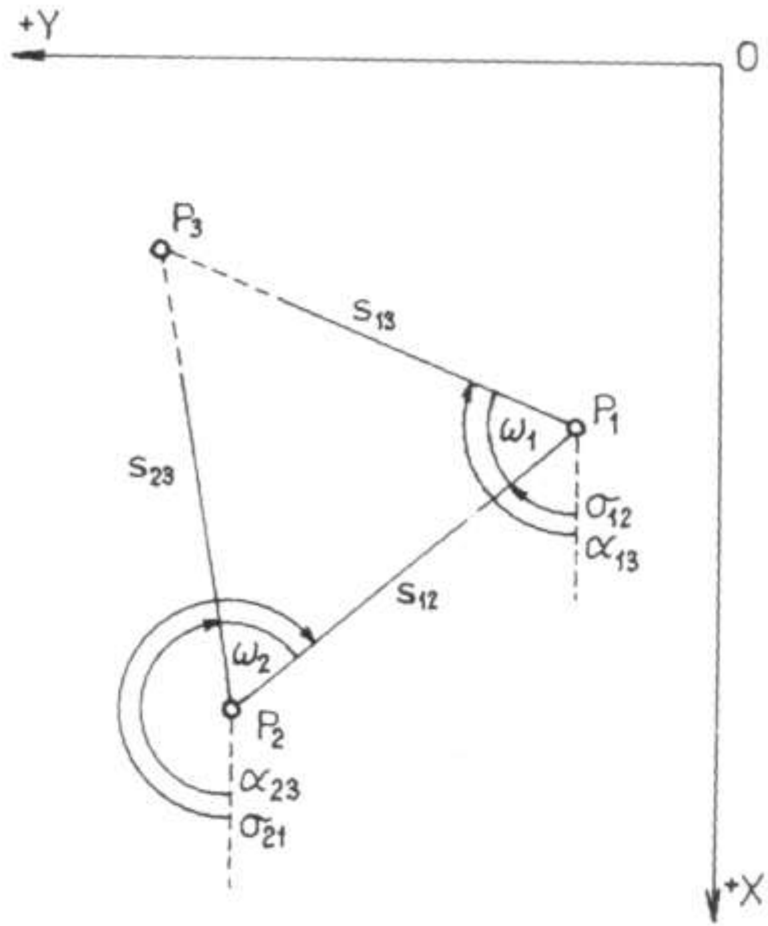
$\Delta y_{13} = d_{13} . \sin \alpha_{13}$

$\Delta x_{13} = d_{13} . \cos \alpha_{13}$

$y_3 = y_1 + \Delta y_{13} = y_1 + d_{13} . \sin \alpha_{13}$

$x_3 = x_1 + \Delta x_{13} = x_1 + d_{13} . \cos \alpha_{13}$

Calculation of the coordinates by intersection from angles



Given:

rectangular coordinates of points P_1 [y_1, x_1] and P_2 [y_2, x_2],

horizontal angles ω_1 a ω_2

Calculated: P_3 [y_3, x_3]

$$tg\ \varphi_{12} = \frac{|\Delta y_{12}|}{|\Delta x_{12}|}$$

according to the table $\rightarrow \sigma_{12}$

$\sigma_{21} = \sigma_{12} + 200$ gon

$$s_{12} = \sqrt{\Delta x_{12}^2 + \Delta y_{12}^2}$$

$s_{13} = s_{12} \,.\, \sin \omega_2 \,/\, \sin (200 \text{ gon} - (\omega_1 + \omega_2)) =$

$s_{12} \,.\, \sin \omega_2 \,/\, \sin (\omega_1 + \omega_2)$,

$s_{23} = s_{12} \,.\, \sin \omega_1 \,/\, \sin (200 \text{ gon} - (\omega_1 + \omega_2)) =$

$s_{12} \,.\, \sin \omega_1 \,/\, \sin (\omega_1 + \omega_2)$ (law of sines)

$\alpha_{13} = \sigma_{12} + \omega_1$

$\alpha_{23} = \sigma_{21} - \omega_2$

$y_3 = y_1 + s_{13} . \sin \alpha_{13} = y_2 + s_{23} . \sin \alpha_{23}$

$x_3 = x_1 + s_{13} . \cos \alpha_{13} = x_2 + s_{23} . \cos \alpha_{23}$

Coordinates of the point P_3 are determined twice using bearings and distances to check the calculation.

Intersection from distances

Given:

rectangular coordinates of points P_1 [y_1, x_1] and P_2 [y_2, x_2],

measured horizontal distances d_{13} a d_{23}

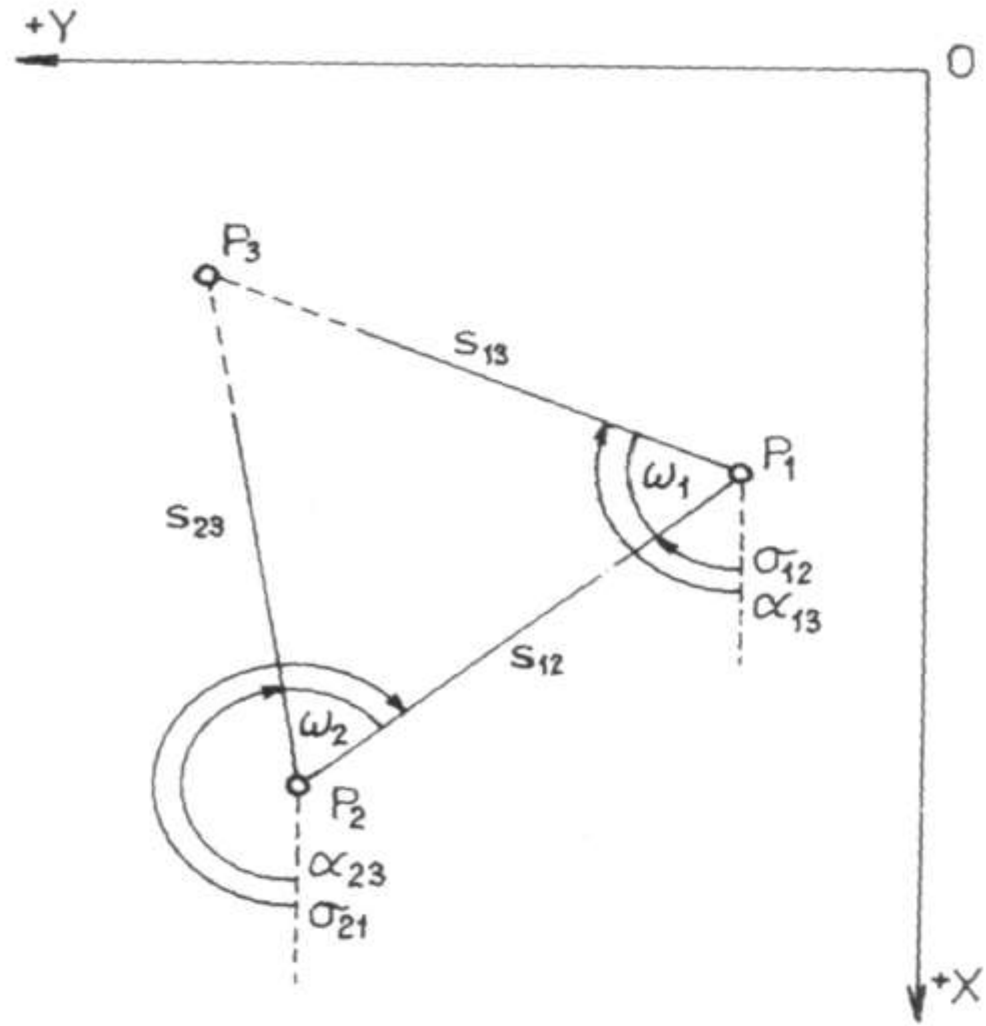
Calculated: rectangular coordinates of P_3 [y_3, x_3]

$$\sigma_{21} = \sigma_{12} + 200 \text{ gon}$$

$$s_{12} = \sqrt{\Delta x_{12}^2 + \Delta y_{12}^2}$$

$$\cos(\omega_1) = \frac{s_{13}^2 + s_{12}^2 - s_{23}^2}{2 \cdot s_{13} \cdot s_{12}}$$

$$\cos(\omega_2) = \frac{s_{23}^2 + s_{12}^2 - s_{13}^2}{2 \cdot s_{23} \cdot s_{12}}$$



$$\alpha_{13} = \sigma_{12} + \omega_1$$

$$\alpha_{23} = \sigma_{21} - \omega_2$$

$$y_3 = y_1 + s_{13} \,.\, \sin \alpha_{13} = y_2 + s_{23} \,.\, \sin \alpha_{23}$$

$$x_3 = x_1 + s_{13} \,.\, \cos \alpha_{13} = x_2 + s_{23} \,.\, \cos \alpha_{23}$$

Coordinates of the point P_3 are determined twice using bearings and distances to check the calculation.

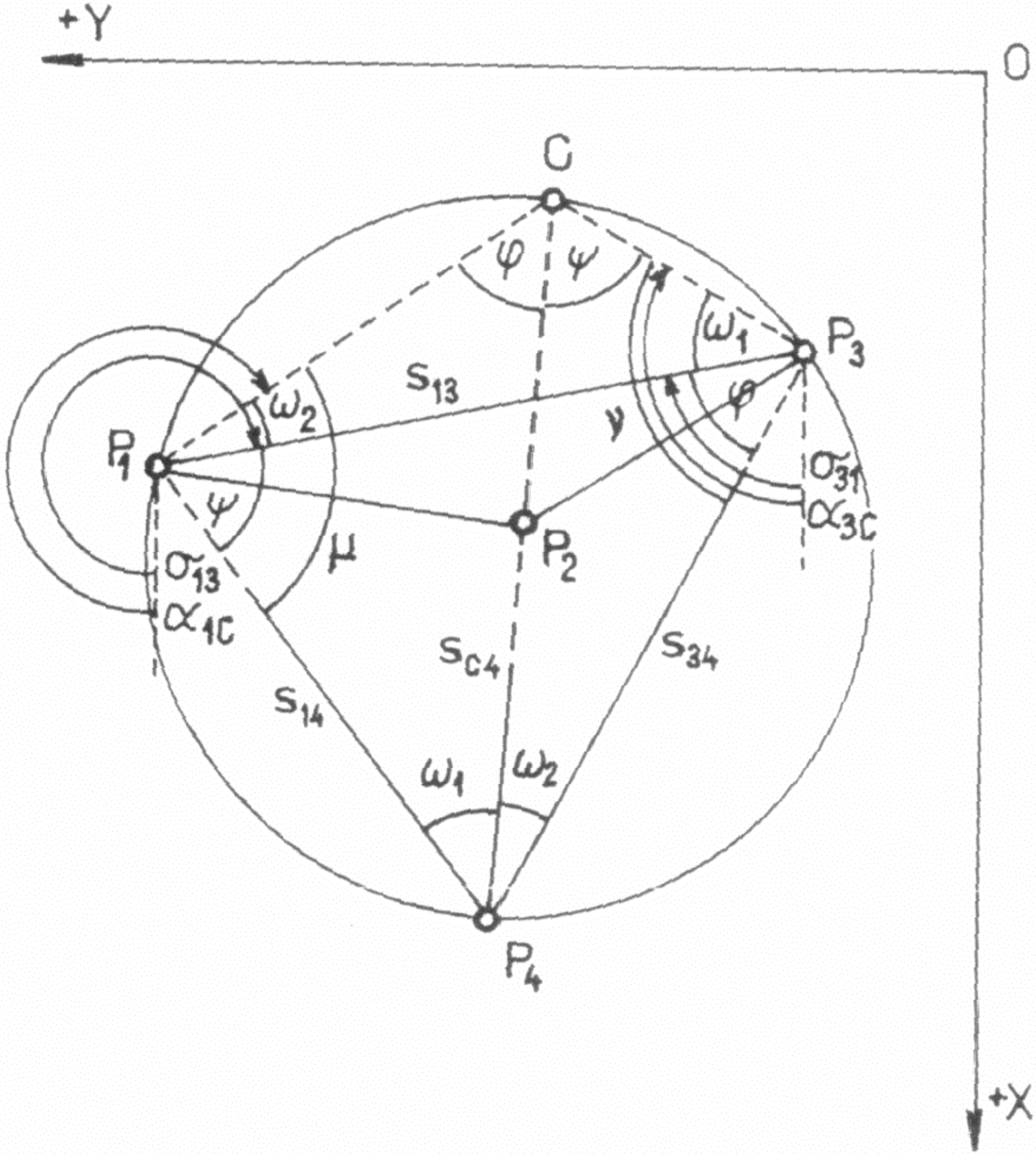
Resection

Given:

rectangular coordinates of points P_1 [y_1, x_1], P_2 [y_2, x_2], P_3 [y_3, x_3]

measured horizontal angles ω_1 a ω_2

Calculated: rectangular coordinates of P_4 [y_4, x_4]


+Y
O
C
φ
ψ
ω1
P3
ω2
s13
P1
y
φ
ψ
σ31
α3C
P2
μ
σ13
α1C
sc4
s34
s14
ω1
ω2
P4
+X

Traverse (polygon)

- a broken line connecting two survey points
- traverse points = vertexes of the broken line
- traverse legs = joins of nearby traverse points
- horizontal angles at all traverse points and lengths of traverse legs are measured
- coordinates Y, X of the traverse points are calculated

Traverse

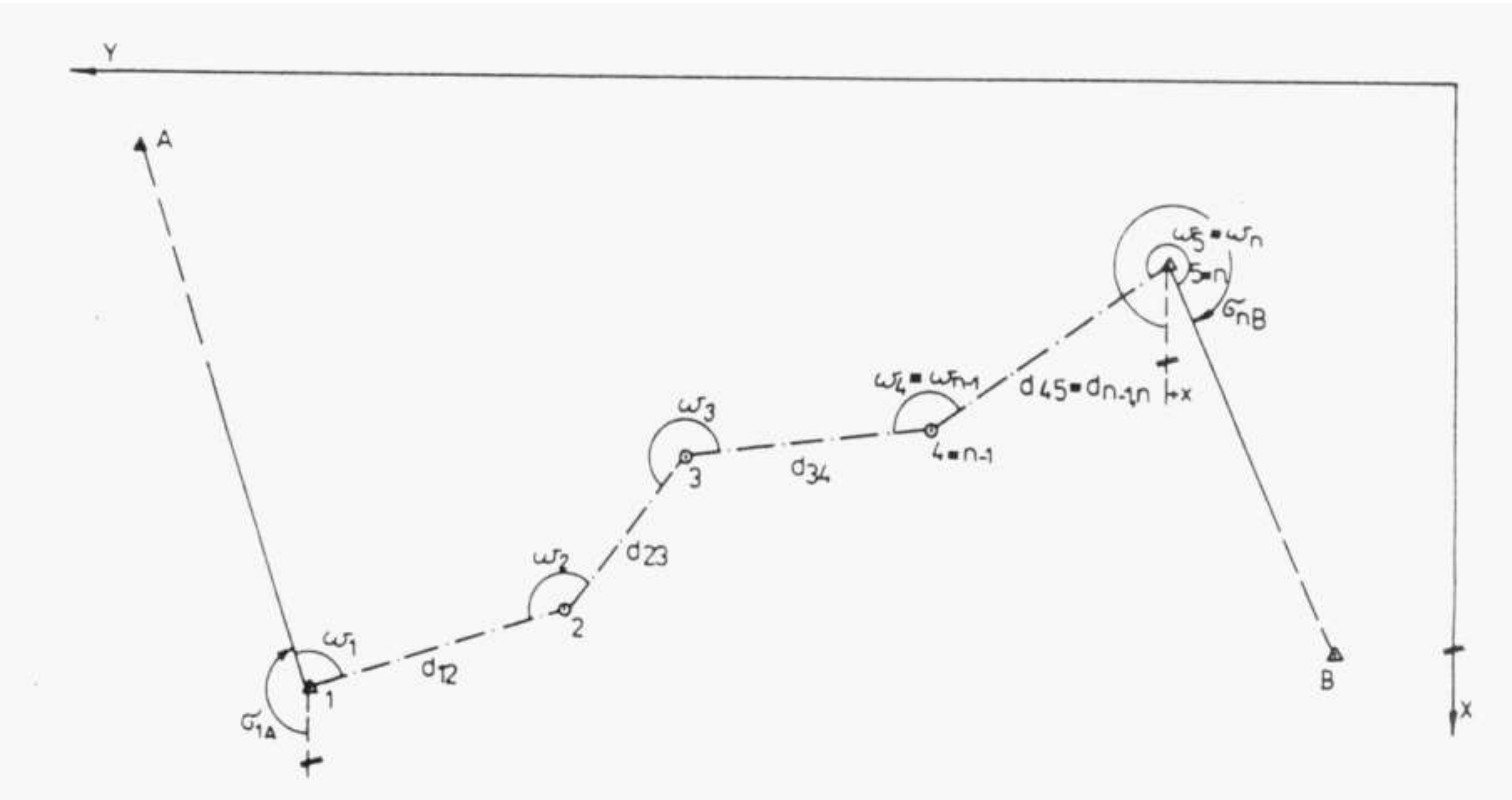
- **connected** (at one or both ends)
  – the traverse is connected to the survey points whose coordinates are known
- **disconnected** – the traverse is connected to the survey points whose coordinates are not known

Dividing traverses according to a shape:

- **traverse line**
- **closed traverse** – the start point = the end point

Orientation of a traverse = measurement of the horizontal angle at the start (or the end) point.

Traverse connected and oriented on both ends



Given:

coordinates of the start and the end points

1 [y_1, x_1], n [y_n, x_n] (here n = 5)

coordinates of the orientation points A [y_A, x_A],

B [y_B, x_B]

measured horizontal distances d_{12}, d_{23}, d_{34}, d_{45}

measured horizontal angles ω_1, ω_2, ω_3, ω_4, ω_5

Calculated:

coordinates of points 2 [y_2, x_2], 3 [y_3, x_3], …,

n-1 [y_{n-1}, x_{n-1}]

1. calculation of bearings

$$tg \quad \varphi_{1A} = \frac{|\Delta y_{1A}|}{|\Delta x_{1A}|}$$

$$tg \quad \varphi_{nB} = \frac{|\Delta y_{nB}|}{|\Delta x_{nB}|}$$

according to the table $\rightarrow$ σ_{1A} and σ_{nB}

2. angular adjustment

Angular error O_ω is calculated (error = „it should be" minus „it is". „It should be" is the bearing σ_{nB} calculated from coordinates, „it is" is the bearing α_{nB} calculated using measured horizontal angles).

$$O_\omega = \sigma_{nB} - (\sigma_{1A} + \sum \omega_i - (n - 1).200^g)$$

i = 1, … , n

n … number of the traverse points (here n = 5)

Clause for the angular adjustment:

$$|O_\omega| \leq u_{M\omega}$$

$$u_{M\omega} = 0,01^g \sqrt{n+3}$$

The angular error is divided equally to the measured horizontal angles:

$\delta_\omega = O_\omega / n$

$\omega'_1 = \omega_1 + \delta_\omega$, ... , $\omega'_n = \omega_n + \delta_\omega$.

3. calculation of bearings

$$\alpha_{12} = \sigma_{1A} + \omega'_{1}$$

$$\alpha_{23} = \alpha_{12} + \omega'_{2} \pm 200^{g}$$

…

$$\alpha_{n-1,n} = \alpha_{n-2,n-1} + \omega'_{n-1} \pm 200^{g}$$

$$\alpha_{nB} = \alpha_{n-1,n} + \omega'_{n} \pm 200^{g} = \sigma_{nB} \quad \text{Check!}$$

4. calculation of coordinate differences

$\Delta y_{12} = d_{12} \,.\, \sin \alpha_{12}$

…

$\Delta y_{n-1,n} = d_{n-1,n} \,.\, \sin \alpha_{n-1,n}$

$\Delta x_{12} = d_{12} \,.\, \cos \alpha_{12}$

…

$\Delta x_{n-1,n} = d_{n-1,n} \,.\, \cos \alpha_{n-1,n}$

5. calculation of coordinate deviations

$$\Delta y_{1n} = y_n - y_1$$

$$\Delta x_{1n} = x_n - x_1$$

$$\Delta y_{1n}^{cal} = \Delta y_{12} + \Delta y_{23} + \Delta y_{34} + \Delta y_{4n} = \Sigma \Delta y$$

$$\Delta x_{1n}^{cal} = \Delta x_{12} + \Delta x_{23} + \Delta x_{34} + \Delta x_{4n} = \Sigma \Delta x$$

$$O_y = \Delta y_{1n} - \Sigma \Delta y$$

$$O_x = \Delta x_{1n} - \Sigma \Delta x$$

Positional difference

$$O_p = \sqrt{O_x^2 + O_y^2}$$

Clause for the adjustment:

$$O_p \leq u_{Mp}$$

$$u_{Mp} = 0{,}01 \sqrt{\sum d} + 0{,}1$$

Corrections of coordinate differences

$$\delta_{\Delta y_{ij}} = \frac{O_y}{\sum |\Delta y|} |\Delta y_{ij}|$$

$$\delta_{\Delta x_{ij}} = \frac{O_x}{\sum |\Delta x|} |\Delta x_{ij}|$$

The corrections of coordinate differences are not equal, they depend on values of coordinate differences.

6. corrected coordinate differences

$$\Delta y'_{12} = \Delta y_{12} + \delta_{\Delta y_{12}} \qquad \Delta x'_{12} = \Delta x_{12} + \delta_{\Delta x_{12}}$$

…

$$\Delta y'_{n-1,n} = \Delta y_{n-1,n} + \delta_{\Delta y_{n-1,n}} \qquad \Delta x'_{n-1,n} = \Delta x_{n-1,n} + \delta_{\Delta x_{n-1,n}}$$

$$\sum \Delta y' = \Delta y_{1n} \qquad \sum \Delta x' = \Delta x_{1n}$$

Check!

7. calculation of adjusted coordinates

y_1 = given x_1 = given

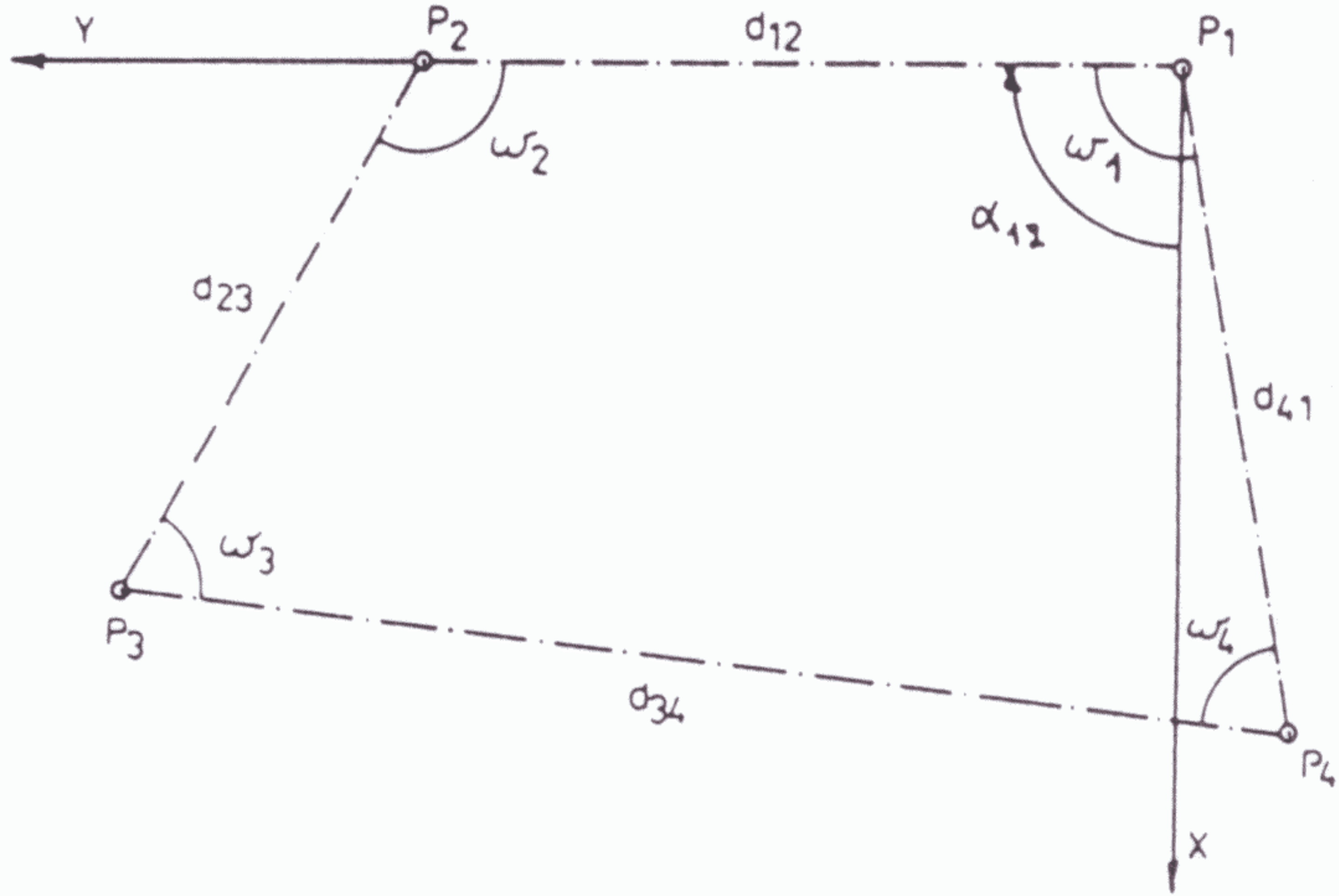
$y_2 = y_1 + \Delta y'_{12}$ $x_2 = x_1 + \Delta x'_{12}$

….

$y_n = y_{n-1} + \Delta y'_{n-1,n}$ = given Check!

$x_n = x_{n-1} + \Delta x'_{n-1,n}$ = given Check!

Closed traverse without orientation



Given:

measured horizontal distances d_{12}, d_{23}, d_{34}, d_{41}

measured horizontal angles ω_1, ω_2, ω_3, ω_4

Calculated:

coordinates of points P_1 [y_1, x_1], P_2 [y_2, x_2],
P_3 [y_3, x_3], P_4 [y_4, x_4]

1. choice of a local coordinate system

One of the traverse points is chosen as a beginning of a local coordinate system (here P_1) and one axis is put in the traverse leg from this point (here axis +Y is put in P_1P_2). Coordinates of the beginning are chosen, usually:

$y_1 = 0{,}00$, $x_1 = 0{,}00$

Result from this choice:

$x_2 = 0{,}00$, $\sigma_{12} = 100^g$

The calculation is the same as previous one, the start point = the end point = P_1.

2. angular adjustments

$$O_\omega = (n - 2).200 - \sum \omega_i$$

i = 1, … , n

n … number of the traverse points (here n = 4)

Clause for the angular adjustment:

$$|O_\omega| \leq u_{M\omega}$$

$$u_{M\omega} = 0,01^g \sqrt{n+3}$$

Angular error is divided equally to the measured horizontal angles:

δ_ω = O_ω / n

$\omega'_1 = \omega_1 + \delta_\omega$, ... , $\omega'_n = \omega_n + \delta_\omega$.

3. calculation of bearings

$\alpha_{12} = \sigma_{12} = 100^g$

$\alpha_{23} = \alpha_{12} + \omega'_2 \pm 200^g$

...

$\alpha_{41} = \alpha_{34} + \omega'_4 \pm 200^g$

$\alpha_{12} = \alpha_{41} + \omega'_1 \pm 200^g = \sigma_{12}$ Check!

4. calculation of coordinate differences

$\Delta y_{12} = d_{12} \,.\, \sin \alpha_{12}$

…

$\Delta y_{41} = d_{41} \,.\, \sin \alpha_{41}$

$\Delta x_{12} = d_{12} \,.\, \cos \alpha_{12}$

…

$\Delta x_{41} = d_{41} \,.\, \cos \alpha_{41}$

5. calculation of coordinate deviations

$$\Delta y_{1n} = y_n - y_1 = 0$$

$$\Delta x_{1n} = x_n - x_1 = 0$$

$$\Delta y_{1n}^{cal} = \Delta y_{12} + \Delta y_{23} + \Delta y_{34} + \Delta y_{4n} = \Sigma \Delta y$$

$$\Delta x_{1n}^{cal} = \Delta x_{12} + \Delta x_{23} + \Delta x_{34} + \Delta x_{4n} = \Sigma \Delta x$$

$$O_y = - \Sigma \Delta y$$

$$O_x = - \Sigma \Delta x$$

Positional difference

$$O_p = \sqrt{O_x^2 + O_y^2}$$

Clause for the adjustment:

$$O_p \leq u_{Mp}$$

$$u_{Mp} = 0{,}01 \sqrt{\sum d} + 0{,}1$$

Corrections of coordinate differences

$$\delta_{\Delta y_{ij}} = \frac{O_y}{\sum|\Delta y|} |\Delta y_{ij}|$$

$$\delta_{\Delta x_{ij}} = \frac{O_x}{\sum|\Delta x|} |\Delta x_{ij}|$$

6. corrected coordinate differences

$$\Delta y'_{12} = \Delta y_{12} + \delta_{\Delta y_{12}} \qquad \Delta x'_{12} = \Delta x_{12} + \delta_{\Delta x_{12}}$$

....

$$\Delta y'_{41} = \Delta y_{41} + \delta_{\Delta y_{41}} \qquad \Delta x'_{41} = \Delta x_{41} + \delta_{\Delta x_{41}}$$

$$\sum \Delta y' = 0 \qquad \sum \Delta x' = 0$$

Check!

7. calculation of adjusted coordinates

y_1 = given $\quad$ x_1 = given

$y_2 = y_1 + \Delta y'_{12}$ $\quad$ $x_2 = x_1 + \Delta x'_{12}$

….

$y_1 = y_4 + \Delta y'_{41}$ = given Check!

$x_1 = x_4 + \Delta x'_{41}$ = given Check!

Thank you for your attention!